A PORTMANTEAU TEST FOR CORRELATION IN SHORT PANELS

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Abstract

Inoue and Solon (2006) presented a test against serial correlation of arbitrary form in fixed-effect models for short panel data. Implementing the test requires choosing a regularization parameter that may severely affect power and for which no optimal selection rule is available. We present a modified version of their test that does not require any regularization parameter. Asymptotic power calculations illustrate the improvement of our procedure. An extension of the approach that accommodates dynamic models is also provided.

Keywords: asymptotic power, fixed effects, short panel data, serial correlation, specification test.

JEL classification: C12, C23, C55.

Introduction

Inoue and Solon (2006) proposed a test against serial correlation of arbitrary form in linear fixed-effect models for short panel data. This portmanteau approach is desirable if no strong stand can be taken on the particular form of correlation that should serve as the alternative. This is relevant in many panel data applications, especially when the observations for a given unit do not have a natural ordering such as time. Tests against specific alternatives have been discussed by Baltagi and Li (1995), Baltagi and Wu (1999), and Wooldridge (2002, p. 282–283) and Drukker (2003).

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The test of Inoue and Solon (2006) involves the choice of a regularization parameter. This choice affects the power of the test, possibly quite dramatically. The way in which it does, and to what extent, depends on the particular alternative under consideration, and this in a quite convoluted way. Examples are given below. Choosing the regularization parameter, therefore, amounts to taking a stand on what type of alternative one wishes to best arm oneself against. This is at odds with the portmanteau paradigm that underlies the test.

We show in this paper that working with this regularization parameter is both inefficient and unnecessary, and we provide a modified test statistic. Local asymptotic power against MA(1) and AR(1) alternatives is calculated in three-period data to illustrate the gains that can be obtained by using the modified test statistic. We also present a generalized version of our test procedure that can be applied to models with predetermined regressors, such as lagged dependent variables.

1 Portmanteau tests for serial correlation

Consider $N \times T$ panel data where, for each unit $i = 1, \dots, N$, we observe the outcome vector $\boldsymbol{y}_i = (y_{i1}, \dots, y_{iT})'$ and the matrix of covariates $\boldsymbol{X}_i = (\boldsymbol{x}_{i1}, \dots, \boldsymbol{x}_{iT})'$. Suppose that

$$\boldsymbol{y}_i = \boldsymbol{X}_i \boldsymbol{\beta} + c_i \, \boldsymbol{\iota}_T + \boldsymbol{\varepsilon}_i,$$

where c_i is unit i's fixed effect, $\boldsymbol{\iota}_T$ is the T-vector of ones, and $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \dots, \varepsilon_{iT})'$ is a vector of latent disturbances. We will write $\boldsymbol{I}_T$ for the $T \times T$ identity matrix and let $\boldsymbol{M} := \boldsymbol{I}_T - \boldsymbol{\iota}_T (\boldsymbol{\iota}_T' \boldsymbol{\iota}_T)^{-1} \boldsymbol{\iota}_T'$ be the matrix that transforms observations into deviations from within-group means. Everything to follow can be modified to unbalanced panel data in the same way as in Inoue and Solon (2006, p. 841–842).

Inoue and Solon (2006) proposed an elegant portmanteau test for the (composite) null

$$\boldsymbol{\Sigma} := \mathbb{E}(\boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}_i') = \sigma^2 \, \boldsymbol{I}_T, \tag{1.1}$$

where σ^2 is an unknown positive constant, under the following assumption.

Assumption 1.

(a) $\boldsymbol{X}_i$ *and* $\boldsymbol{\varepsilon}_i$ *are i.i.d. and have finite fourth-order moments;*

(b) $\mathbb{E}(\boldsymbol{\varepsilon}_i|\boldsymbol{X}_i, c_i) = \boldsymbol{0}$*;*

(c) $\mathbb{E}(\boldsymbol{X}_i'\boldsymbol{M}\boldsymbol{X}_i)$ *is non-singular;*

(d) $T > 2$*.*

Part (a) is a regularity condition that ensures that a central limit theorem can be used. Parts (b) and (c) imply that $\boldsymbol{\beta}$ is identified and can be estimated by the within-group estimator, which is semiparametrically efficient under the null. Part (d) rules out panels with only two time periods as a portmanteau test of (1.1) cannot be constructed in such a case.

The covariance matrix of $\boldsymbol{e}_i := \boldsymbol{M}\boldsymbol{\varepsilon}_i$ is

$$\boldsymbol{\Omega} := \mathbb{E}(\boldsymbol{e}_i\boldsymbol{e}_i') = \boldsymbol{M}\boldsymbol{\Sigma}\boldsymbol{M}.$$

Under the null,

$$\boldsymbol{\Omega} = \sigma^2\boldsymbol{M}.$$

A natural way to test (1.1) would then be to evaluate whether the difference between an unconstrained and constrained estimator of $\boldsymbol{\Omega}$ can be considered large (in some suitable norm) under the null of no serial correlation. Moreover, we would test the moment condition

$$\mathbb{E}(\boldsymbol{u}_i) = \boldsymbol{0}, \qquad \boldsymbol{u}_i := \operatorname{vech}\left(\boldsymbol{e}_i\boldsymbol{e}_i' - \frac{\boldsymbol{e}_i'\boldsymbol{e}_i}{T-1}\boldsymbol{M}\right), \tag{1.2}$$

recalling that the degrees-of-freedom correction needs to be applied because we have that $\mathbb{E}(\boldsymbol{e}_i'\boldsymbol{e}_i) = \mathbb{E}(\boldsymbol{\varepsilon}_i'\boldsymbol{M}\boldsymbol{\varepsilon}_i) = \sigma^2\operatorname{tr}(\boldsymbol{M}) = \sigma^2(T-1)$.

A test of (1.2) evaluates whether

$$\underline{\operatorname{vech}}(\boldsymbol{\Omega}) = -\boldsymbol{\iota}_{\frac{T(T-1)}{2}}\frac{\sigma^2}{T} \quad \text{and} \quad \operatorname{diag}(\boldsymbol{\Omega}) = \boldsymbol{\iota}_T\left(\sigma^2 - \frac{\sigma^2}{T}\right), \tag{1.3}$$

where we use $\underline{\operatorname{vech}}(\boldsymbol{\Omega})$ to denote the operator that vectorizes the entries of $\boldsymbol{\Omega}$ below its main diagonal. As $\boldsymbol{M}\boldsymbol{\iota}_T = \boldsymbol{0}$, the matrix $\boldsymbol{\Omega}$ is singular and so, too, is the covariance matrix

$$\boldsymbol{V} := \mathbb{E}(\boldsymbol{u}_i\boldsymbol{u}_i').$$

This reflects the fact that some of the moments in (1.3) are redundant. The way forward is to test a linearly-independent subset of the moments. That is, we test

$$\mathbb{E}(\boldsymbol{A}'\boldsymbol{u}_i) = \boldsymbol{0},$$

where $\boldsymbol{A}$ is a (non-stochastic) selection matrix.

For a given selection matrix $\boldsymbol{A}$ the quadratic form

$$s_{\boldsymbol{A}} := (\textstyle\sum_i \boldsymbol{A}'\hat{\boldsymbol{u}}_{\boldsymbol{i}})'(\sum_i \boldsymbol{A}'\hat{\boldsymbol{u}}_{\boldsymbol{i}}\hat{\boldsymbol{u}}_{\boldsymbol{i}}{}'\boldsymbol{A})^{-1}(\sum_i \boldsymbol{A}'\hat{\boldsymbol{u}}_{\boldsymbol{i}}) \tag{1.4}$$

can serve as test statistic for the null. Here, $\hat{\boldsymbol{u}}_i$ is the plug-in estimator of $\boldsymbol{u}_i$ obtained on replacing the unobserved disturbances $\boldsymbol{e}_i$ in (1.2) by the residuals from a within-group regression, i.e., by $\hat{\boldsymbol{e}}_i := \boldsymbol{M}\boldsymbol{y}_i - \boldsymbol{M}\boldsymbol{X}_i\hat{\boldsymbol{\beta}}$, where

$$\hat{\boldsymbol{\beta}} := \left(\textstyle\sum_i \boldsymbol{X}_i'\boldsymbol{M}\boldsymbol{X}_i\right)^{-1}\left(\sum_i \boldsymbol{X}_i'\boldsymbol{M}\boldsymbol{y}_i\right)$$

is the within-group least-squares estimator.

The following theorem summarizes the properties of $s_{\boldsymbol{A}}$ under the null and under Pitman sequences of the form

$$\boldsymbol{\Sigma} = \sigma^2\boldsymbol{I}_T + \frac{\boldsymbol{C}}{\sqrt{N}}$$

for a symmetric matrix $\boldsymbol{C}$ with zero main diagonal. Under such a sequence the asymptotic bias in the moment condition equals

$$\boldsymbol{\delta} := \sqrt{N}\,\mathbb{E}(\boldsymbol{u}_i) = \text{vech}\left(\boldsymbol{MCM} + \frac{\boldsymbol{\iota}_T'\boldsymbol{C}\boldsymbol{\iota}_T}{T(T-1)}\,\boldsymbol{M}\right).$$

We let $\chi^2_q(\delta)$ denote the non-central chi-squared distribution with q degrees of freedom and non-centrality parameter δ.

Theorem 1. *Let Assumption 1 hold. Suppose that $\boldsymbol{A}'\boldsymbol{V}\boldsymbol{A}$ is non-singular for a chosen $T(T+1)/2 \times q$ selection matrix $\boldsymbol{A}$.*

(a) Under the null (1.1), $s_{\boldsymbol{A}} \xrightarrow{d} \chi^2_q(0)$.

(b) Under a sequence of local alternatives $s_{\boldsymbol{A}} \xrightarrow{d} \chi^2_q(\delta_{\boldsymbol{A}})$, where $\delta_{\boldsymbol{A}} := \boldsymbol{\delta}'\boldsymbol{A}(\boldsymbol{A}'\boldsymbol{V}\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{\delta}$.

Proof. We show Part (b); Part (a) follows in the same way by setting $\boldsymbol{\delta} = \mathbf{0}$. Combine the distributional approximation $N^{-1/2} \sum_i \boldsymbol{A}'\hat{\boldsymbol{u}}_i = N^{-1/2} \sum_i \boldsymbol{A}'\boldsymbol{u}_i + o_p(1) \xrightarrow{d} \boldsymbol{N}(\boldsymbol{A}'\boldsymbol{\delta}, \boldsymbol{A}'\boldsymbol{V}\boldsymbol{A})$ with the convergence result $N^{-1} \sum_i (\boldsymbol{A}'\hat{\boldsymbol{u}}_i \hat{\boldsymbol{u}}_i' \boldsymbol{A}) \xrightarrow{p} \boldsymbol{A}'\boldsymbol{V}\boldsymbol{A}$ via Slutsky's theorem to get the result. □

The elements of $\boldsymbol{\delta}$ are all non-zero as soon as two of the off-diagonal entries of the matrix $\boldsymbol{C}$ are different. A consistent test is thus obtained for general choices of the selection matrix. Asymptotic power, however, is sensitive to the choice of $\boldsymbol{A}$, as is apparent from Part (b) of the theorem.

The test statistic proposed by Inoue and Solon (2006) is of the form in (1.4). They set

$$\boldsymbol{A} = \boldsymbol{\Delta}_n := \partial \text{vech}(\boldsymbol{\Omega}) / \partial \underline{\text{vech}}(\boldsymbol{\Omega}_{-n})',$$

where $\boldsymbol{\Omega}_{-n}$ is the $(T-1) \times (T-1)$ submatrix of $\boldsymbol{\Omega}$ obtained by dropping its nth row and column, and n is to be chosen. For a three-period panel, for example, these are the vectors

$$\boldsymbol{\Delta}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \boldsymbol{\Delta}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \boldsymbol{\Delta}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

This leads to as many possible test statistics as there are time periods in the panel. Each of them tests

$$q_0 := \frac{(T-1)(T-2)}{2}$$

restrictions. They are

$$\underline{\text{vech}}(\boldsymbol{\Omega}_{-n}) = -\boldsymbol{\iota}_{q_0} \frac{\sigma^2}{T}.$$

From Part (b) of Theorem 1 the asymptotic power varies with n, in general. It can do so quite substantially; power calculations are performed below to illustrate this. Working with $\boldsymbol{\Delta}_n$ can be motivated by the observation that $\text{rank}(\boldsymbol{\Omega}) = T - 1$ if $\boldsymbol{\Sigma}$ has maximal rank, and so $\boldsymbol{\Omega}_{-n}$ is non-singular.

On the other hand, working with the selection matrix $\boldsymbol{\Delta}_n$ is too conservative. As each row of $\boldsymbol{\Omega}$ sums to zero the restrictions in (1.3) on its diagonal entries are indeed redundant. This leaves the $T(T-1)/2$ restrictions on the off-diagonal entries of $\boldsymbol{\Omega}$. It is easy to see, however, that

$$q_1 := \frac{T(T-1)}{2} - 1$$

of these are non-redundant. Observe that $q_1 - q_0 = T - 2$, so that the test of Inoue and Solon (2006) ignores information for any choice of n and for all T.

A natural way to proceed, then, is to test all q_1 non-redundant moments simultaneously. The choice of which entry of $\underline{\text{vech}}(\boldsymbol{\Omega})$ to drop is arbitrary; any choice will deliver the same test statistic. Here, we drop the lower-left entry, $(\boldsymbol{\Omega})_{T,1}$. This corresponds to working with

$$\boldsymbol{A} = \boldsymbol{\Delta} := \partial \text{vech}(\boldsymbol{\Omega}) / \partial \underline{\text{vech}}^*(\boldsymbol{\Omega})',$$

for $\underline{\text{vech}}^*(\boldsymbol{\Omega})$ the operator that returns all lower-diagonal entries of $\boldsymbol{\Omega}$ except for $(\boldsymbol{\Omega})_{T,1}$. With three periods, $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_1, \boldsymbol{\Delta}_3)$. Then testing (1.3) is equivalent to testing the q_1 moments $\mathbb{E}(\boldsymbol{\Delta}' \boldsymbol{u}_i) = \boldsymbol{0}$, and is done by using

$$s_{\boldsymbol{\Delta}} = (\textstyle\sum_i \boldsymbol{\Delta}' \hat{\boldsymbol{u}}_i)' (\sum_i \boldsymbol{\Delta}' \hat{\boldsymbol{u}}_i \hat{\boldsymbol{u}}_i{}' \boldsymbol{\Delta})^{-1} (\sum_i \boldsymbol{\Delta}' \hat{\boldsymbol{u}}_i).$$

This test evaluates whether

$$\underline{\text{vech}}^*(\boldsymbol{\Omega}) = -\boldsymbol{\iota}_{q_1} \frac{\sigma^2}{T}$$

holds and involves no choice of regularization parameter.

We observe that a numerically-equivalent way of constructing the test statistic $s_{\boldsymbol{\Delta}}$ is as

$$(\textstyle\sum_i \hat{\boldsymbol{u}}_i)' (\sum_i \hat{\boldsymbol{u}}_i \hat{\boldsymbol{u}}_i{}')^* (\sum_i \hat{\boldsymbol{u}}_i),$$

where $\boldsymbol{V}^*$ denotes the Moore-Penrose pseudo-inverse of matrix $\boldsymbol{V}$. Theorem 1 can equally be shown to hold for this form of our test statistic, relying on Andrews (1987, Theorem 1).

2 Power calculations

We compare the original Inoue and Solon (2006) test with our proposal by means of asymptotic-power calculations for three-period data.

First consider covariance-stationary MA(1) processes,

$$\varepsilon_{it} = \eta_{it} + \theta\, \eta_{it-1}, \qquad \eta_{it} \sim \text{white noise}\,(0, \sigma^2),$$

for finite θ. Here,

$$\boldsymbol{\Sigma} = \sigma^2 \begin{pmatrix} 1+\theta^2 & \theta & 0 \\ \theta & 1+\theta^2 & \theta \\ 0 & \theta & 1+\theta^2 \end{pmatrix},$$

and the null corresponds to $\theta = 0$.

The test of Inoue and Solon (2006) tests a single moment condition. For $n = 1, 2, 3$ it is

$$(\boldsymbol{\Omega})_{3,2} = -\frac{\sigma^2}{3}, \qquad (\boldsymbol{\Omega})_{1,3} = -\frac{\sigma^2}{3}, \qquad (\boldsymbol{\Omega})_{1,2} = -\frac{\sigma^2}{3},$$

respectively. After some algebra the non-centrality parameter in Part (b) of Theorem 1 associated with each of these tests is found to be

$$\delta_{\boldsymbol{\Delta}_1} = \frac{\theta^2}{9}, \qquad \delta_{\boldsymbol{\Delta}_2} = 4\,\frac{\theta^2}{9}, \qquad \delta_{\boldsymbol{\Delta}_3} = \frac{\theta^2}{9},$$

so that setting $n = 2$ would maximize power here. Note that this implies that a test on the second-order autocovariance (of the demeaned errors) is more powerful than a test on the first-order autocovariance, even though serial correlation in the (original) disturbances is only present at the first order. This finding conflicts with the advice given in Inoue and Solon (2006, p. 841) that a test that sets $n = 1$ or $n = T$ should be used in cases where serial correlation is most pronounced at first order.

With three periods our approach tests two autocovariances. We find

$$\delta_{\boldsymbol{\Delta}} = 4\,\frac{\theta^2}{9},$$

which coincides with the maximum of the non-centrality parameters of the individual tests. Of course, the relevant limit distribution here features two degrees of freedom as opposed to just one, so that our test will be less powerful than the most powerful of this univariate tests.

The second class of alternatives we look at consists of the stationary AR(1) processes,

$$\varepsilon_{it} = \rho\,\varepsilon_{it-1} + \eta_{it}, \qquad \eta_{it} \sim \text{white noise}\,(0, \sigma^2),$$

for $\rho \in (-1, 1)$. In this case,

$$\boldsymbol{\Sigma} = \frac{\sigma^2}{1-\rho^2} \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{pmatrix},$$

and the null is achieved when $\rho = 0$. We calculate

$$\delta_{\boldsymbol{\Delta}_1} = \frac{\varrho^2}{9}, \qquad \delta_{\boldsymbol{\Delta}_2} = 4\,\frac{\varrho^2}{9}, \qquad \delta_{\boldsymbol{\Delta}_3} = \frac{\varrho^2}{9},$$

where $\varrho := \rho/(1+\rho)$, for the three possible univariate tests, and

$$\delta_{\boldsymbol{\Delta}} = 4\,\frac{\varrho^2}{9}$$

for the joint test. These parameters are as in the MA(1) case, with ϱ replacing θ, and so the main conclusions reached there carry over.

In Figure 1 we provide the power functions for our bivariate test (solid line), for the univariate test with maximal power (dashed-dotted line), and for the two other univariate tests (dashed line) for a sample of size 100. The plots clearly show a large power gain of our procedure relative to the univariate tests with $n \in \{1, 3\}$, and only a minor power loss relative to the (optimal) univariate test with $n = 2$.

3 Testing with predetermined regressors

The test statistic above corresponds to the score statistic in a model with normal errors. The original derivation of Inoue and Solon (2006) was motivated as such. The formulation of the null as in (1.2) highlights that the procedure remains valid in the absence of normality. It further suggests that a similar test statistic can be derived when Assumption 1 (b)–(c) is relaxed to allow for models where the regressors are predetermined as opposed to strictly exogenous.

Figure 1: Power comparisons

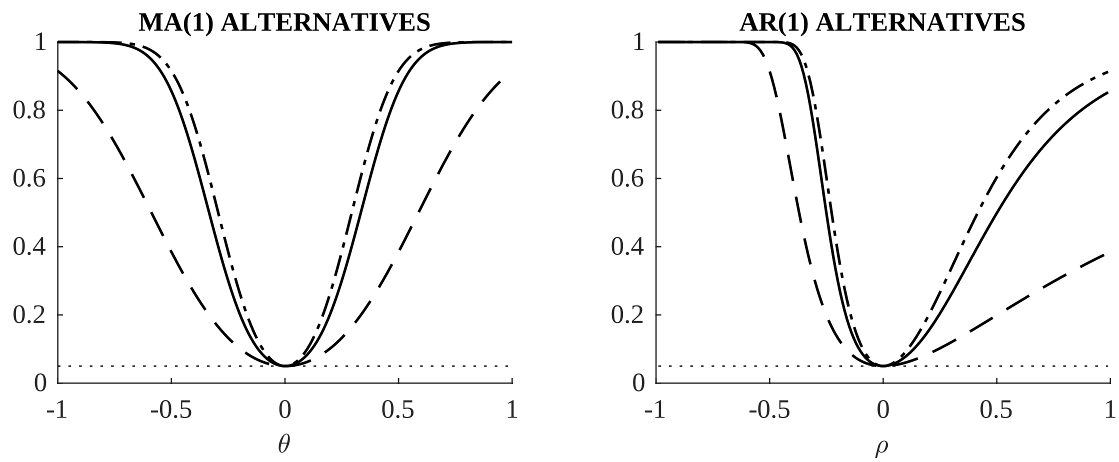

Figure notes. Power functions for the Inoue and Solon (2006) tests (dashed and dashed-dotted lines) and for our modified test (solid line) for a 100×3 panel. The size of each of the tests is 5% (horizontal dotted line).

Moreover, suppose that an estimator $\check{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ can be constructed that satisfies

$$\sqrt{N}(\check{\boldsymbol{\beta}} - \boldsymbol{\beta} - \mathbb{E}(\boldsymbol{\varphi}_i)) = \sum_i \frac{\boldsymbol{\varphi}_i - \mathbb{E}(\boldsymbol{\varphi}_i)}{\sqrt{N}} + o_p(1),$$

where $\mathbb{E}(\boldsymbol{\varphi}_i) = \mathbf{0}$ under the null, $\mathbb{E}(\boldsymbol{\varphi}_i) = \boldsymbol{\gamma}/\sqrt{N}$ for some finite $\boldsymbol{\gamma}$ under local alternatives, and $\mathbb{E}(\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i')$ exists. Generalized method-of-moment estimators such as those given in Ahn and Schmidt (1995) would be a natural choice. For autoregressions likelihood-based estimators such as those in Dhaene and Jochmans (2016) can be an attractive alternative as they will preserve the fact that the test statistic is invariant to the scale of the fixed effects.

If we let

$$\check{\boldsymbol{u}}_i := \operatorname{vech}\left(\check{\boldsymbol{e}}_i\check{\boldsymbol{e}}_i' - \frac{\check{\boldsymbol{e}}_i'\check{\boldsymbol{e}}_i}{T-1}\boldsymbol{M}\right), \qquad \check{\boldsymbol{e}}_i := \boldsymbol{M}\boldsymbol{y}_i - \boldsymbol{M}\boldsymbol{X}_i\check{\boldsymbol{\beta}},$$

and introduce the matrix of derivatives $\boldsymbol{\Gamma} := \mathbb{E}\left(\partial \boldsymbol{u}_i/\partial\boldsymbol{\beta}'\right)$, then standard arguments yield

$$\sum_i \frac{\check{\boldsymbol{u}}_i}{\sqrt{N}} = \sum_i \frac{\boldsymbol{u}_i + \boldsymbol{\Gamma}\boldsymbol{\varphi}_i}{\sqrt{N}} + o_p(1).$$

This motivates the use of the modified test statistic

$$\check{s}_{\boldsymbol{A}} := (\textstyle\sum_i \boldsymbol{A}'\check{\boldsymbol{u}}_i)'(\sum_i \boldsymbol{A}'(\check{\boldsymbol{u}}_i + \check{\boldsymbol{\Gamma}}\check{\varphi}_i)(\check{\boldsymbol{u}}_i + \check{\boldsymbol{\Gamma}}\check{\varphi}_i)'\boldsymbol{A})^{-1}(\sum_i \boldsymbol{A}'\check{\boldsymbol{u}}_i),$$

where we let $\check{\boldsymbol{\varphi}}_i$ and $\check{\boldsymbol{\Gamma}}$ denote estimators of $\boldsymbol{\varphi}_i$ and $\boldsymbol{\Gamma}$. If $N^{-1}\sum_i \|\check{\boldsymbol{\varphi}}_i - \boldsymbol{\varphi}_i\|^2 = o_p(1)$ then Theorem 1 will continue to go through for this modified test statistic subject to redefining

$$\delta_{\boldsymbol{A}} := (\boldsymbol{\delta} + \boldsymbol{\Gamma}\boldsymbol{\gamma})' \boldsymbol{A}(\boldsymbol{A}'\boldsymbol{V}\boldsymbol{A})^{-1}\boldsymbol{A}'(\boldsymbol{\delta} + \boldsymbol{\Gamma}\boldsymbol{\gamma})$$

and $\boldsymbol{V} := \mathbb{E}((\boldsymbol{u}_i + \boldsymbol{\Gamma}\boldsymbol{\varphi}_i)(\boldsymbol{u}_i + \boldsymbol{\Gamma}\boldsymbol{\varphi}_i)')$. The matrix $\boldsymbol{\Gamma}$ will generally be non-zero unless Assumption 1 (b) holds (in which case Theorem 1 goes through without modification). The vector $\boldsymbol{\gamma}$ will be non-zero when $\check{\boldsymbol{\beta}}$ is asymptotically-biased under the sequence of local alternatives to the null that is being considered. This will be the case quite generally in models with predetermined regressors.

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